Frequency Distributions of Energy Deposition by Fast Charged Particles in Very Small Pathlengths*

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The frequency distribution of energy losses in an equimolar mixture of He-CO_2 by 46.4-MeV protons has been examined experimentally and compared with the distributions predicted by the Blunck-Leisegang–corrected Vavilov theory. The distributions studied correspond to mean energy losses of 1.56, 7.85, and 15.6 keV. The theoretical predictions of the distributions with the larger mean energy losses agree well with the experimental data in the region about the most probable energy loss. However, agreement between the values of the measured and predicted full widths at half-maximum of the distribution found for a mean energy loss of 1.56 keV is not as good as for the other distributions. This is due partly to a larger uncertainty in the experimental parameters of the calculation. Considerable disagreement exists when theory and experiment are compared in regions of high and low energy losses for all of the distributions.

I. INTRODUCTION

It has previously been shown[1] that when the full width at half-maximum (FWHM) is used as a basis of comparison, the Blunck-Leisegang–corrected[2] Vavilov[3] distribution agrees quite well with experiment for fast protons (43.7 MeV). Comparison of the experimental distributions with the theory over a substantial range of energy losses was not made.

We have now generated the theoretical distribution functions using the tables given by Seltzer and Berger[4] and the method of Fano[5] for applying the Blunck-Leisegang correction to such distributions. The values given by Seltzer and Berger[4] were extended for large energy losses by using the values of the Landau distribution given in the tables published by Börsch-Supan.[6] This is permissible since for small values of the Vavilov[3] parameter κ the Vavilov distribution goes smoothly into the Landau[7] distribution. In calculating the corrected distributions, use is made of a parameter designated by Blunck and Leisegang[2] as b^2. This parameter is directly proportional to the second moment of the resonance cross-section distribution and inversely proportional to the pathlength over which the particle loses energy. Blunck and Leisegang[2] gave a general method for its calculation. In a later paper, Blunck and Westphal[8] gave an alternative representation for b^2 based on a Thomas-Fermi model of the atom. For fast particles suffering small energy losses, discrepancies between these two calculations of b^2 on the order of 10–20% appear. The more exact method of calculation causes the previous agreement[1] between the predicted and experimental FWHM's for the shortest pathlength to disappear.

New data have been generated so that good counting statistics were available over a very broad range of energy losses. Because of a slight change in the experimental setup from that described in Ref. 1, the energy of the protons entering our counter was increased and the proton energy for the data reported here is 46.4 ± 0.05 MeV.

The causes of and the factors which control the statistical spread of the energy-loss distribution function have been discussed by Hilbert *et al.*[1] The value of ϵ_{max} (the maximum energy that can be lost by the proton in a single collision with an electron) for 46.4-MeV protons is 103.5 keV. The values of the various parameters pertinent to the present set of experiments are given in Table I.

II. EXPERIMENTAL PROCEDURE

The experimental setup was similar to that previously used[1] with the exception that a shorter air path for the beam was used in the present work. This increased the energy of the protons entering the counter from 43.7 to 46.4 MeV. The same gas flow proportional counter was used for the energy-loss measurements. Some changes were

TABLE I. Various parameters pertinent to energy-loss distribution functions of 46.4-MeV protons.

Pathlength (g/cm^2)	Mean energy loss (keV)	κ	b^2	$\xi = \kappa\epsilon_{max}$ (keV)
1.33×10^{-4}	1.56	0.00114	40.5	0.111
6.66×10^{-4}	7.85	0.00530	8.15	0.552
1.33×10^{-3}	15.6	0.00990	4.40	1.022

also made in our electronics. The output of the amplifying chain in the proportional counter line was fed to the input of a linear gate. The signal from the p-i-n detector was passed through a single-channel analyzer (window ~40–50 MeV) and the subsequent logic signal was used to open a linear gate for approximately 10 μsec. This time was sufficiently long to accommodate the time constants of the proportional counter signal (2-μsec first differentiation; 2-μsec integration). The linear signal was then processed by a multichannel pulse-height analyzer.

Spectral distortion due to pile up was carefully looked for by changing the counting rate and comparing the spectra collected. All data were taken at very low counting rates ($<10^3$ counts/min). This appears to be at least an order of magnitude less than the rate where spectral distortion appears.

III. THEORETICAL COMPUTATIONS

The Blunck-Leisegang–corrected[2] Vavilov[3] distribution can be expressed as[5]

$$f(X,\Delta)=\frac{1}{(\sqrt{\pi})b\xi}\int_{-\infty}^{+\infty} f_\nu(X,\Delta')e^{\frac{-(\Delta-\Delta')^2}{\xi^2 b^2}}\,d\Delta', \quad (1)$$

where $f_\nu(X,\Delta')$ is the Vavilov distribution, b is the Blunck-Leisegang parameter,

$$\xi = \frac{2\pi N_0 z^2 e^4}{mv^2}\,\frac{Z}{A}X = 0.154\,\frac{z^2}{\beta^2}\,\frac{Z}{A}X\ \text{MeV}, \quad (2)$$

N_0 is Avagadro's Number, X is the pathlength (g/cm^2), Z is the atomic number of medium, A is the atomic weight of medium, v equals βc which is the velocity of incident particle, and z is the charge of incident particle. The expression for the energy loss given in terms of the Landau parameter λ is

$$\Delta=\xi\lambda+\overline{\Delta}+\xi\,(1+\beta^2-\gamma+\ln\kappa). \quad (3)$$

Here $\overline{\Delta}$ is average energy lost in pathlength X, γ is Euler's constant which is 0.577216,

$$\kappa = \text{Vavilov parameter} = \xi/\epsilon_{\max}, \quad (4)$$

and $\epsilon_{\max}$ is the maximum energy transfer to an electron in a single collision.

The function $f(X,\Delta)$ was evaluated numerically. The range of values for λ (−11.0–48.0) used was great enough to prevent an artificial cutoff of the resultant distribution function in the region of interest.

Because of the difficulty with inverting large matrices, the experimental resolution was folded into the theoretical curves rather than unfolded from the experimental curves. This was accomplished by applying a Gaussian probability function to each point of the Blunck-Leisegang–corrected Vavilov[3] distribution. This broadening function had three components and was assumed to be Gaussian. The broadening due to the statistics of ion-pair formation and electron multiplication at the counter wire can be shown to be given by[9]

$$\sigma_{\text{stat}} = 0.169E^{1/2}, \quad (5)$$

where E is the energy deposited by the proton in the sensitive volume of the proportional counter. The noise broadening was determined by measuring the FWHM of a pulse supplied by a mercury relay pulser. The resolution was determined by measuring the FWHM of the energy-loss distribution produced by the α particles emitted from a thin collimated α source (Am^{241}, $E_\alpha = 5.48$ MeV). Since both resolution (counter) and noise broadening are functions of both the operating voltage of the counter and the electronic gains of the preamplifier and amplifier systems used, each component was determined for each of the three experimental operating conditions. The values of these quantities are given in Table II. The three σ's were then combined for each channel in the distribution by taking the square root of the sum of the squares to yield a σ for the total broadening due to experimental factors. The resultant distributions then represent the theoretical distributions corrected for the experimentally introduced distortions (electronic noise, electron and multiplication statistics, and inherent counter resolution). The probability function used was

$$P(\Delta,\Delta')=1/\sqrt{2\pi}\sigma(\Delta')\exp[-(\Delta-\Delta')^2/2\sigma^2(\Delta')]. \quad (6)$$

The distribution function compared to that obtained from the experiments was then given by

$$F(X,\Delta)=\sum_{\Delta'}P(\Delta,\Delta')f(X,\Delta'). \quad (7)$$

IV. RESULTS

Both the experimental data and the theoretical distribution functions are shown in Figs. 1–3. These have been normalized to unit probability. The ordinates are, therefore, expressed as probability per unit energy loss and the abscissas as energy loss per event. A semilogarithmic scale has been chosen to illustrate more graphically those regions where discrepancies between the-

TABLE II. Broadening of experimental spectra due to noise and inherent counter resolution.

Pathlength (g/cm^2)	σ_{noise}[a] (FWHM)	σ_{res} (% FWHM)
1.33×10^{-4}	2.2 channels	7.5
6.66×10^{-4}	0.9 channels	3.6
1.33×10^{-3}	0.6 channels	3.9

[a]FWHM when experimental spectrum is collected in 100 channels.

ory and experiment have appeared. The solid lines represent the computed distribution functions and the circles, the experimental data. The error bars show only the statistical uncertainties. In the case of the shortest pathlength (Fig. 1), the statistical uncertainty was less than the size of the circles and error bars were, therefore, omitted. We attribute the lack of smoothness of our data to the long counting times used, which accentuates electronic instabilities. However, these are not large enough to obscure the obvious differences between theory and experiment.

In order to obtain an absolute scale (average energy loss from stopping-power theory equal to average channel numbers) for the experimental curves, the following procedure was used: In each case, it was observed that when the logarithm of the number of events in each channel was plotted against channel number, the high-energy tail portion could be approximated by a straight line. A least-squares fitting process was then used to determine the slope and the tail extrapolated to find the channel corresponding to one event. Using this point as the high-energy cutoff, the mean channel number of the distribution was then found and set equal to the mean energy loss as calculated from the stopping power.

V. DISCUSSION

Although other investigations have examined the degrees of agreement between theory and experiment, the pathlengths considered were definitely greater than those used in our experiments. Maccabee *et al.*,[10] in addition to other particles, used protons of essentially the same energy as we did and compared his experimental data with calculated Vavilov distributions for energy losses incurred by the passage of the protons through thin silicon detectors. This resulted in a κ equaling 2.23, which is some two orders of magnitude greater than our case. Agreement was attained. However, using 730-MeV protons ($\kappa = 0.0055$), discrepancies appeared. As pointed out by these authors the loss of secondaries may account for these at small energy losses; however, this would cause a larger discrepancy to appear in the region of high energy losses than is shown from their results and our findings.

Bichsel[11] has noted that an improvement in the formulation of a Blunck-Leisegang-type correction can be obtained by incorporating into the theory a quantum-mechanical treatment of the effects

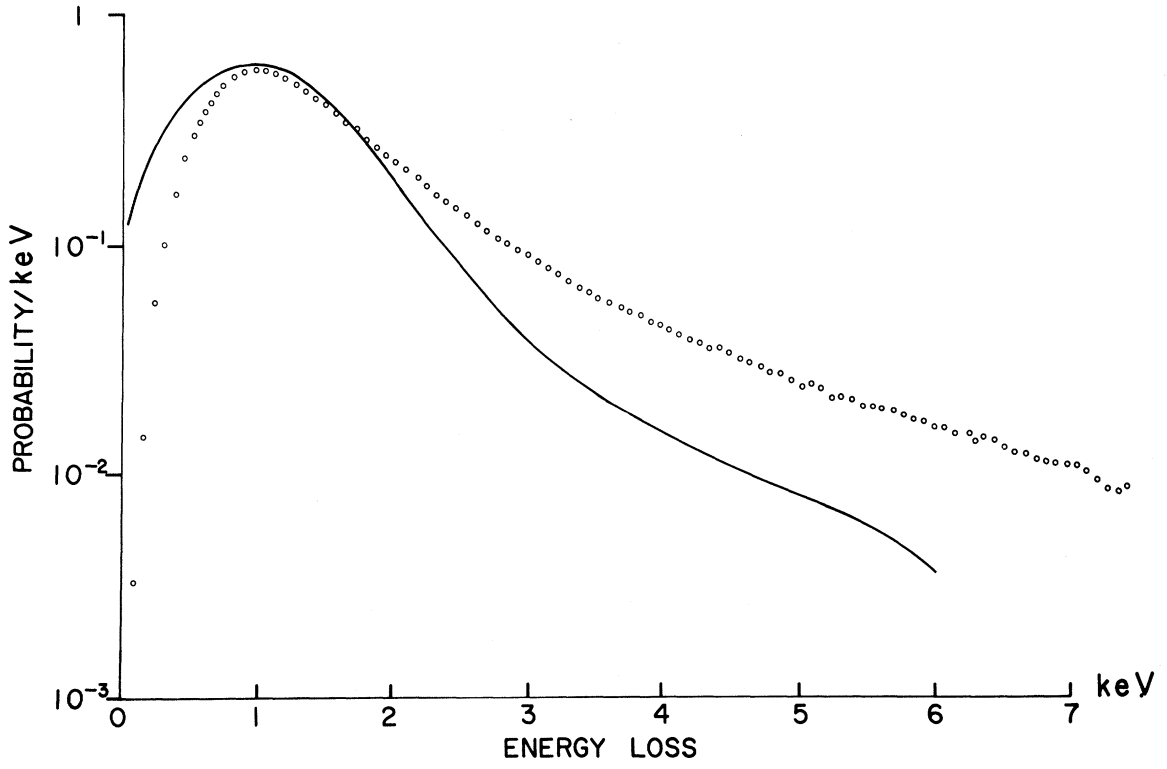

FIG. 1. Frequency distribution of energy losses for 46.4-MeV protons in equimolar mixture of He-CO_2 in a pathlength of 1.33×10^{-4} g/cm^2. Circles show experimental data, while the solid line is fully corrected theoretical function.

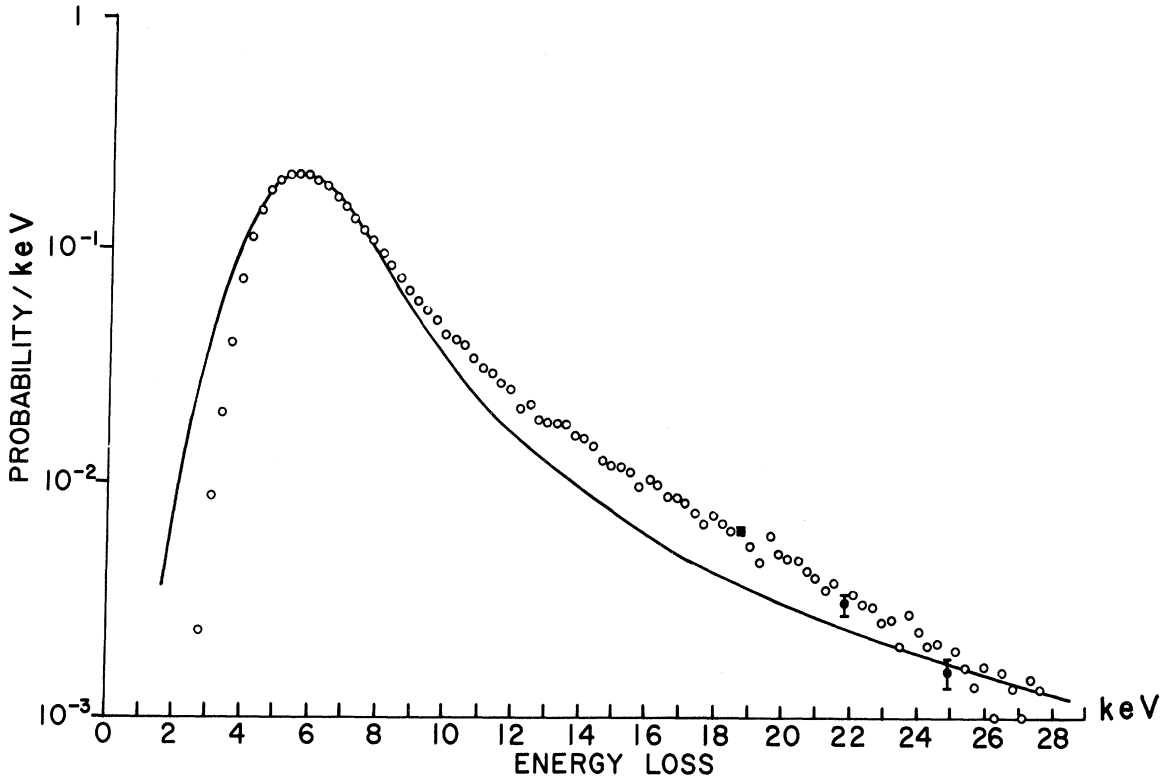

FIG. 2. Frequency distribution of energy losses for 46.4-MeV protons in equimolar mixture of He-CO_2 in a pathlength of 6.66×10^{-4} g/cm^2. Circles show experimental data, while the solid line is fully corrected theoretical function.

due to the binding of the atomic electrons.

The experimental distribution functions show definite discrepancies with those computed from theory. First, the theory predicts a greater number of low-energy events than were found. Second, it predicts less high-energy events than were obtained experimentally. Such distortion in the experimental spectra could be due to pulse pile up. However, we have carefully investigated this possibility by accumulating spectra at different counting rates. We feel that at the counting rates used, pile up did not occur and that there is a real discrepancy between experiment and theory. A real distortion of the experimental spectra exists because of the loss of secondaries from the sensitive volume. This loss would subtract events from the high-energy end and add to the low-energy portion. This contribution if accounted for would, therefore, increase the discrepancy found for the high-energy event portion of the various distributions.

A possible reason for these discrepancies could result from the application of a Gaussian function as the folding function used to correct the Vavilov distribution. It might be noted that the correction function only reduces to a Gaussian shape through the neglect of the third and higher moments of the resonance cross section ($\langle Kr^3\rangle_{av}$, $\langle Kr^4\rangle_{av}$, ...). The standard deviation of this Gaussian function is $\xi b/\sqrt{2}$. One would expect significant contributions to the energy-loss distributions for the region where $\Delta\sim0$, when the most probable energy loss is approximately given by $(3\xi b'/\sqrt{2}) = \bar{\Delta} + \xi\,(1+\beta^2 + \ln\kappa - 0.577\,216)$, i.e., $(\lambda\sim0)$. Here, b' is a parameter which gives the width of a trial Gaussian correction function. The ratio of b'/b which is proportional to $\xi\ (E_{mp}/\bar{\Delta})$, where E_{mp} is the most probable energy loss, increases as the pathlength increases. Thus, one would expect better agreement between theory and experiment, particularly for small energy losses, as the pathlength is increased. This is, in fact, found experimentally.

To raise the theoretical distribution in the region of high energy loss, one needs to employ a folding function with a larger third moment than is characteristic of a Gaussian. If this folding function also rises to its maximum value faster than a Gaussian, the theoretical distribution in the low energy-loss region would be brought closer to the experimental values. Whether a folding function would actually result from inclusion of the third moment $\langle Kr^3\rangle_{av}$ has not been investigated.

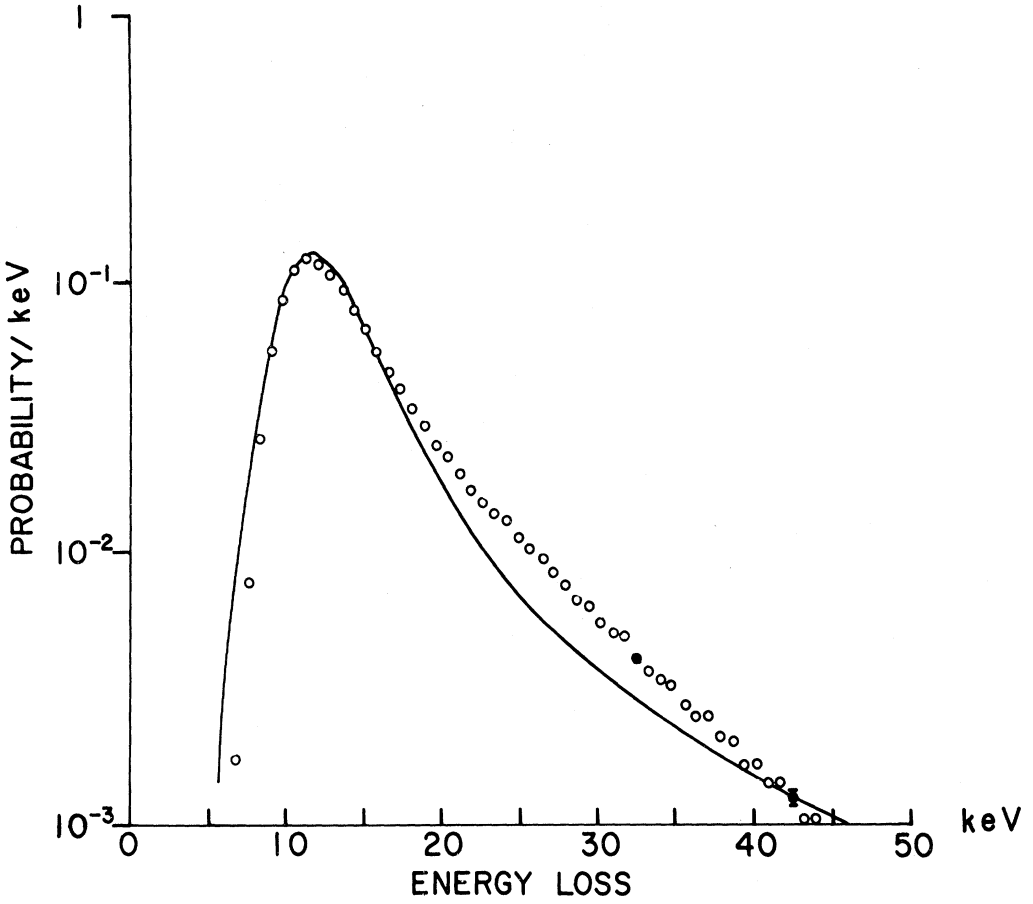

FIG. 3. Frequency distribution of energy losses for 46.4-MeV protons in equimolar mixture of He-CO_2 in a pathlength of 1.33×10^{-3} g/cm^2. Circles show experimental data, while the solid line is fully corrected theoretical function.

The low energy-loss region of the theoretical distribution can be brought closer to the experimental values by arbitrarily choosing an empirical value of b^2. However, there does not appear to be a b^2 that will fit the distribution over its entire range. This indicates that the nonagreement between theory and experiment is not simply due to the approximation[2] in the calculation of the second moment of the resonance cross section $\langle Kr^2\rangle_{av}$. The discrepancies between theory and experiment do not seem to be due to approximations made in the evaluation of the experimental or theoretical parameters, but appear to be inherent in the method used to make the correction.

TABLE III. Comparison of theoretical and experimental frequency distribution functions for values close to most probable energy loss.

Pathlength (g/cm^2)	FWHM (keV) Expt	FWHM (keV) Theory	FWHM/E_{mp} (%) Expt	FWHM/E_{mp} (%) Theory
1.33×10^{-4}	1.29	1.46	132	150
6.66×10^{-4}	3.96	3.94	72.0	71.6
1.33×10^{-3}	6.4	6.3	55.7	54.3

Another discrepancy in the theory appears in the region of the most probable energy loss for the shortest pathlength used. This is shown in Table III. In the previous paper, an approximation for the value of b^2 was used,[8] and good agreement was found between theory and experiment in this region. The method originally proposed by Blunck and Leisegang[2] for the calculation of b^2 produced a substantially different set of values. The values computed by each method are given in Table IV. By using the method originally suggested by Blunck and Leisegang[2] for computing the value of the parameter b^2, a broader distribution with an FWHM in good agreement with experiment for the two longer pathlengths is reached. However, the value computed in this manner predicts a broader distribution for the shortest pathlength used. This discrepancy is not apparent in calcu-

TABLE IV. Values of b^2 computed by the two methods suggested (Refs. 2 and 8).

Computed according to Blunck and Leisegang (Ref. 2)	Computed according to Blunck and Westphal (Ref. 8)	Ratio
40.5	34.0	1.19
8.15	6.88	1.18
4.40	3.98	1.10

lating distributions for the longer pathlengths.

VI. CONCLUSION

Although fair agreement between experiment and the Blunck-Leisegang-corrected[2] Vavilov[3] theory for the frequency distribution of small energy losses by fast charged particles in low atomic number materials has been found when the FWHM has been used for comparison, a rather poor fit results when comparisons are made over the entire range of energy losses. The discrepancies between this theory and our experimental data appear to increase as the mean energy loss for the distribution relative to the particle energy decreases.

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[1]J. W. Hilbert, N. A. Baily, and R. G. Lane, Phys. Rev. 168, 290 (1968).
[2]O. Blunck and S. Leisegang, Z. Physik 128, 500 (1950).
[3]P. V. Vavilov, Zh. Eksperim. i Teor. Fiz. 32, 320 (1957) [Soviet Phys. JETP 5, 749 (1957)].
[4]S. M. Seltzer and M. J. Berger, Natl. Acad. Sci. Natl. Res. Council, Publ. 1133, 187 (1944).
[5]U. Fano, Ann. Rev. Nucl. Sci. 13, 1 (1963).
[6]W. Börsch-Supan, J. Res. Natl. Bur. Std. (U. S.), 65B, 245 (1961).
[7]L. Landau, J. Phys. USSR 8, 201 (1944).
[8]O. Blunck and K. Westphal, Z. Physik 130, 641 (1951).
[9]J. L. Campbell and D. K. Ledingham, Brit. J. Appl. Phys. 17, 769 (1966).
[10]H. D. Maccabee, M. R. Raju, and C. A. Tobias, Phys. Rev. 165, 469 (1968).
[11]H. Bichsel, Phys. Rev. B 1, 2854 (1970).

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Fast-Electron Channeling Investigated by Means of Rutherford Scattering

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The Rutherford-scattering and transmission yields for electrons incident on thin single crystals have been measured. For the Rutherford-scattering yield, very pronounced peaks are found whenever the incoming beam is incident on the crystal within a certain critical angle of a low-index direction. The full width at half-maximum for the peak in yield is found to be proportional to Ψ_1, the Lindhard critical angle. Also, the volume of the peak is found to be conserved for gold crystals of thicknesses up to around 5000 Å. Further, experimental investigations of the multiple scattering for single crystals and polycrystalline foils are discussed.

INTRODUCTION

In the past few years, the influence of lattice structure on the motion of heavy charged particles in crystals has been widely investigated (for a review, see Ref. 1). It has been shown that the experimental results are in good agreement with Lindhard's theory[2] derived from classical orbital mechanics. It has also been shown that these directional effects are powerful tools for the localization of foreign atoms in single crystals and for measurements of radiation damage in single crystals and its annealing behavior. Until now, investigations of directional effects by means of light